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DISTINCTIVE FEATURES OF THE INTRACHAMBER INSTABILITY OF COMBUSTION IN LIQUID-PROPELLANT ROCKET ENGINES

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Self-oscillations and certain of their regularities determined by solution of a degenerate system of differential equations that is used in considering combustion instability in combustion chambers of liquid-propellant rocket engines are modeled mathematically.

Introduction. The nonstationarity of the process of burning of a fuel in different devices is generated by the chemical-kinetic, diffusion-thermal, and convective phenomena [1]. Furthermore, according to the first low of thermodynamics, the heat released in combustion of the fuel in the flow is converted to the internal energy and components of the total head whose value is dependent on the flow rate. The necessary condition of excitation of self-oscillations in the case in question is the presence of the ascending branch on the head characteristic [2, 3], which is formed by the heat-to-head conversion.

In [4], it has been established that, in burning of the fuel in solid-fuel engines, the pressure dependence of the rate of formation of the gas can be such that the system will lose stability [5]. In liquid-propellant rocket engines, the condition of intrachamber instability lies in the formation of the ascending branch of the $p_{\text{c.ch}} = f(G)$ dependence of the pressure in the combustion chamber on the flow rate. Growth in the head or the pressure $p_{\text{c.ch}}$ in the combustion chamber with flow rate features prominently among the causes of excitation of vibrational-combustion self-oscillations; it has not been considered in theoretical description of the phenomenon of instability of combustion in liquid-propellant rocket engines. The reason is that the conditions of formation of the ascending branch on the $p_{\text{c.ch}} = f(G)$ dependence of the head characteristic of the combustion chamber on the flow rate remained unknown.

Thus, e.g., in [5, 6], the above dependence of the head characteristic has been represented as being monotonically decreasing. In [5], the stability of the stationary regime with such a dependence has been substantiated. Therefore, combustion instability in liquid-propellant rocket engines was mainly determined by the mechanism of phenomenological lag of the combustion of the fuel [5–7] and by the manifestation of different feedbacks which are internal and act universally.

Formulation of the Problem. In [2, 3], it has been established that with decrease in the wave resistance $Z = \sqrt{L_{\text{a,c.ch}}/C_{\text{a,c.ch}}}$ harmonic oscillations due to the heat supply change to relaxation ones whose amplitudes, at a certain Z^*, are independent of further decrease in Z values for $-Z < Z^*$. Such self-oscillations self-excited in the model of vertical primary furnaces of industrial units [3] and oscillations in the well-known Higgins phenomenon of "singing" flames in a Riecke tube [2] are not related to the Crocco mechanism in view of the constancy of their amplitude and the independence of the limiting cycle from the lag τ [7]. In [2, 3], the $C_{\text{a,c.ch}}$ values of the accumulator of mass of the oscillatory circuit have been increased to reduce the wave resistance.

We seek to establish, by mathematical modeling, the distinctive features of relaxation self-oscillations formed in liquid-propellant rocket engines on reduction in the $L_{\text{a,c.ch}}$ to zero. The system (presented below) of equations of nonstationary motion on condition that $L_{\text{a,c.ch}} \to 0$ becomes a system consisting of differential and algebraic equations which were used in considering combustion instability in [5], since the differential equation of pulse balance [8] changes to an algebraic dependence.

System of Equations of Intrachamber Combustion Instability. Taking into account that the mass of combustion products is $m = \rho_{\text{c.ch}} ls$, we can write the pulse-balance equation $d(mw) = (p_{\text{f}} - h_{\text{fr}} - p_{\text{fr}})s dt$ [8] as follows:

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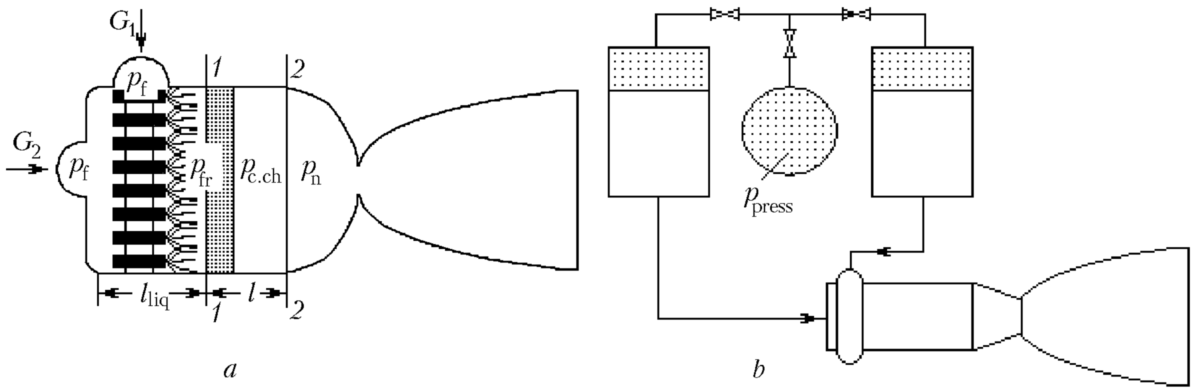

Fig. 1. Diagrams of the considered combustion chamber of a liquid-propellant rocket engine (a) and the pressure system of propellant feeding (b), in which the tanks are pressurized by compressed gas at a pressure p_{press}.

$$L_{\text{a,c.ch}} \frac{dG}{dt} = p_{\text{f}} - h_{\text{fr}}(G) - p_{\text{fr}}, \quad L_{\text{a,c.ch}} = \frac{l_{\text{liq}}}{s}. \tag{1}$$

The mass equation in the combustion chamber is represented, according to [5], in the form

$$\frac{dm}{dt} = G(t - \tau) - G_{\text{n}},$$

where

$$G = G_{\text{pr}} + G_{\text{ox}}, \quad G_{\text{n}} = s_{\text{cr}} \beta \frac{p_{\text{n}}}{c(T_{\text{n}})}, \text{ and } \beta = k \left(\frac{2}{k+1} \right)^{(k+1)/2(k-1)}.$$

Since $dp_{\text{c.ch}}/d\rho_{\text{c.ch}}$ is equal to c^2, it may be reduced to the form

$$C_{\text{a,c.ch}} \frac{dp_{\text{n}}}{dt} = G(t - \tau) - s_{\text{cr}} \beta \frac{p_{\text{n}}}{c(T_{\text{n}})}, \quad C_{\text{a,c.ch}} = \frac{V_{\text{c.ch}}}{c^2}. \tag{2}$$

The characteristic of the nozzle of a liquid-propellant rocket engine $p_{\text{n}} = \varphi(G_{\text{n}}, c(T_{\text{n}}))$ is determined in the following manner: $p_{\text{n}} = c(T_{\text{n}})G_{\text{n}}/(s_{\text{cr}}\beta)$; the dependence $T_{\text{n}}(p_{\text{n}})$ is assumed to be increasing, which has also been taken in [5].

According to the first law of thermodynamics, the energy equation for the flux between the cross sections *1–1* and *2–2* (Fig. 1a) can be written as follows:

$$q + E_1 = E_2 + \Delta h_T + \Delta h_l \qquad \text{or} \qquad q - u_2 - u_1 = \frac{p_{\text{n}}}{\rho_{\text{c.ch}}} - \frac{p_{\text{fr}}}{\rho_{\text{c.ch}}} + \Delta h_T + \Delta h_l.$$

Considering the supply of combustion heat to be isobaric in which $q = c_p(T_{\text{c.ch}} - T_{\text{liq}})$ and taking into account that $\Delta u = u_2 - u_1 = c_v(T_{\text{c.ch}} - T_{\text{liq}})$ and $c_p - c_v = R$, we represent the energy equation in the form

$$\rho_{\text{c.ch}} R (T_{\text{c.ch}} - T_{\text{liq}}) = p_{\text{n}} - p_{\text{fr}} + h_T(G) + h_l(G), \tag{3}$$

in which $h_T(F) = \rho_{\text{c.ch}} \Delta h_T$ and $h_l(F) = \rho_{\text{c.ch}} \Delta h_l$.

Eliminating the variable p_{fr} determined from dependence (3) in Eq. (1), we obtain a system of equations describing nonstationary phenomena of vibrational combustion:

$$L_{\text{a,c.ch}} \frac{dG}{dt} = F(G) - p_{\text{n}}, \quad C_{\text{a,c.ch}} \frac{dp_{\text{n}}}{dt} = G(t - \tau) - s_{\text{cr}} \beta \frac{p_{\text{n}}}{c(T_{\text{n}})}, \tag{4}$$

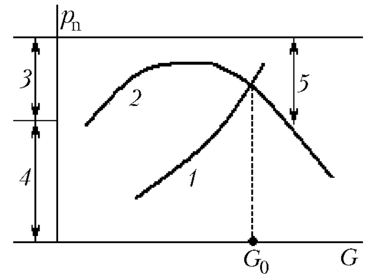

Fig. 2. Determination of the parameters of the stationary regime of the combustion chamber: $p_n = \varphi(G_n,\ c(T_n))$; 2) $F(G)$; 3) $\rho_{c.ch}R(T_{c.ch} - T_{liq})$; 4) p_f; 5) $h(G) = h_{fr}(G) + h_l(G) + h_T(G)$.

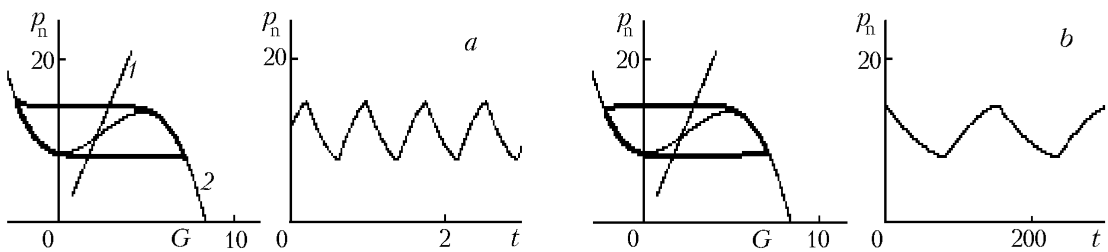

Fig. 3. Modes of relaxation self-oscillations p_c, of constant amplitude for $\tau = 0$, that have been determined on decrease in L_a (a) and on increase in C_a (b): 1) $\varphi(G_n,\ c(T_n))$; 2) $F(G)$. p_n, MPa; G, kg/sec; t, sec.

where $F(G) = p_f + \rho_{c.ch}R(T_{c.ch} - T_{liq}) - h_{fr}(G) - h_T(G) - h_l(G)$ is the head characteristic due to the heat supply and to the pressure of a pressure feeding system p_f (Fig. 1) in which, according to [8], we have $p_{c.ch} = p_f + \rho_{c.ch}R(T_{c.ch} - T_{liq})$.

When $L_{a,c.ch} \to 0$ system (4) becomes degenerate and thereafter a nonlinear equation considered in [5]. Self-oscillations self-excited in such a system are relaxation ones, and the necessary condition for their occurrence at $\tau = 0$ is in the saddle shape of the characteristic $F(G)$. The head characteristic of heat-to-head conversion in the case of phenomena considered in [8] has the ascending branch in the region of small flow rates, whereas for negative flow rates, as in the head characteristics of vane-type blowers, it represents a decreasing dependence in flow rate, which generates its saddle-shaped character. The lag in combustion of the propellant, when $L_{a,c.ch} \to 0$, contributes to the excitation of self-oscillations [5] and increases their amplitude.

For the horizontal arrangement of the combustion chamber, the ascending branch of the characteristic $F(G)$ is formed because of the descending dependence $h_T(G)$ which has the form $h_T(G) = \dfrac{Gw}{2s}\left(1 - \dfrac{T_{c.ch}}{T_{liq}}\right)$ for the isobaric process and is also produced by a number of other factors.

Mathematical Modeling of Vibrational-Combustion Self-Oscillations Due to the Heat Supply in Liquid-Propellant Rocket Engines. The stationary regime of the engine and its parameters are determined by the intersection of the dependences of the following characteristics: the head characteristic $F(G)$ of the combustion chamber and the jet nozzle $p_n = \varphi(G_n,\ c(T_n))$ (Fig. 2). The stationary operating regime of the engine (Fig. 1) is changed by varying the $F(G)$ dependence.

When the operation of a liquid-propellant rocket engine is unstable, self-oscillations are determined by its characteristics, too, but they are additionally dependent on the combustion chamber's wave resistance characterized by the acoustic parameters $L_{a,c.ch}$ and $C_{a,c.ch}$.

It is well known that for $\tau = 0$ the harmonic self-oscillations of a "singing flame" [2, 3], whose amplitudes substantially grow with lag, can be changed by decreasing the wave resistance due to the growth in $C_{a,c.ch}$, to constant-amplitude relaxation oscillations that are absolutely lag-independent. The amplitudes of harmonic oscillations decrease with increase in the wave resistance and then completely disappear. In actual practice, transition to relaxation oscillations whose modes are shown in Fig. 3 is realized by variation of the acoustic parameters; either the quantity

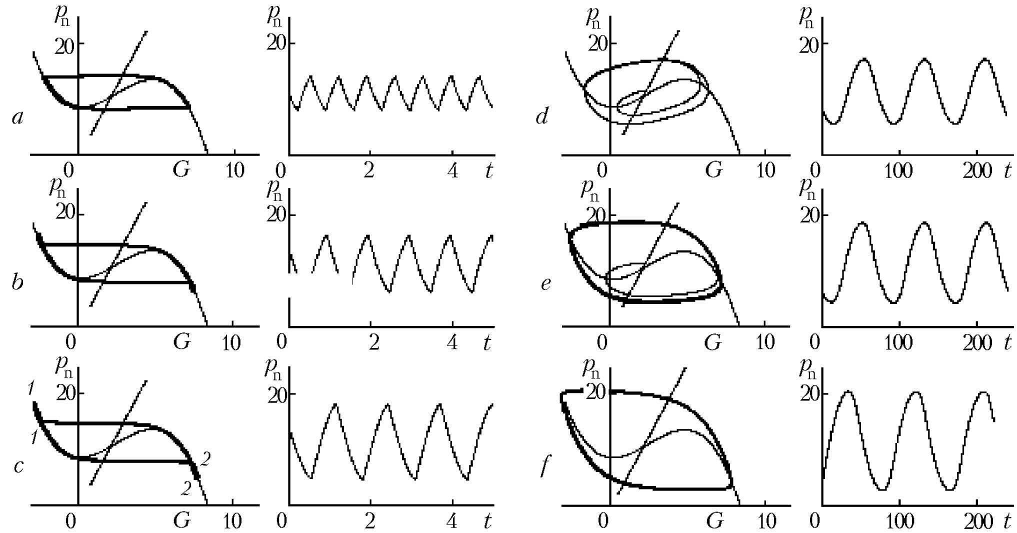

Fig. 4 Changes in the amplitude, frequency, and mode of self-oscillations $p_{\rm n}$ as functions of the increase in the phenomenological lag τ: a–c, relaxation self-oscillations that occur for $L_{\rm a,c.ch} \to 0$; d–f, nearly harmonic oscillations. $p_{\rm n}$, MPa; G, kg/sec; t, sec.

$C_{\rm a,c.ch}$ grows or $L_{\rm a,c.ch}$ decreases, which contributes to the decrease in the wave resistance Z in the cases in question. A distinctive feature of relaxation self-oscillations (Fig. 3) is that their frequency grows with decrease in $L_{\rm a,c.ch}$ and decreases with increase in $C_{\rm a,c.ch}$.

The introduction of the mechanism of lag into the equations of nonstationary combustion of propellant in liquid-propellant rocket engines made it possible to construct the theory of its linear stability but no periodic solutions of the equations of motion have been obtained. Therefore, self-oscillations excited for the phenomenological lag of combustion $\tau \neq 0$ remain to be studied for both a degenerate system [5, 6] and a complete system [8].

The limiting cycles and amplitudes of relaxation self-oscillations (Fig. 3) remain constant, as has been noted above, with increase in the lag τ in the Riecke tube [2, 3]. In the degenerate system of the equations of motion in liquid-propellant rocket engines, when $L_{\rm a,c.ch} \to 0$ and the lag $\tau = 0$, there forms a limiting cycle which is deformed in an unusual manner with increase in $\tau \neq 0$.

This distinctive feature is shown in Fig. 4 where the limiting cycle of relaxation oscillations formed with decrease in $L_{\rm a,c.ch}$, when $\tau = 0$, is represented (Fig. 4a). The oscillation amplitudes grow with the lag τ (Fig. 4b and c) due to the increase in the limiting cycle, which is realized by attachment of the $F(G)$ portions to it: portion *1–1* in the cycle's upper part and portion *2–2* in the lower part (they are singled out in Fig. 4c). The length of these portions of the characteristic grows with τ, which produces the increase in the oscillation amplitude.

The motion of the representation point along the limiting cycle over the oscillation period on portions *1–1* and *2–2* of the characteristic $F(G)$ is carried out twice: in the forward and backward directions.

Figure 4d shows nearly harmonic self-oscillations for $\tau = 0$, whereas Fig. 4e and f shows the increase in their amplitudes and corresponding limiting cycles with τ.

Self-oscillations due to the combustion instability in liquid-propellant rocket engines on condition that $L_{\rm a,c.ch} \to 0$ can be excited, according to [5], on the descending branch of the $p_{\rm c.ch} = F(G)$ plot, when the lag of combustion is $\tau \neq 0$. However, only the boundary of static stability of the working process in the combustion chamber has been determined in [5], but no sufficient conditions for excitation of self-oscillations have been established.

According to [10], the degenerate system of equations for $L_{\rm a,c.ch} \to 0$ has a periodic solution, even if the lag is $\tau = 0$ and the head characteristic is saddle-shaped. This solution represents the mode of relaxation oscillations that occur in the region of the ascending branch of the $p_{\rm c.ch} = F(G)$ plot.

Figure 5 shows the limiting cycle on the descending branch of the characteristic $p_{\rm c.ch} = F(G)$, which is formed only when $\tau \neq 0$ and has the distinctive features considered above. Portions *1–1* and *2–2* of the characteristic

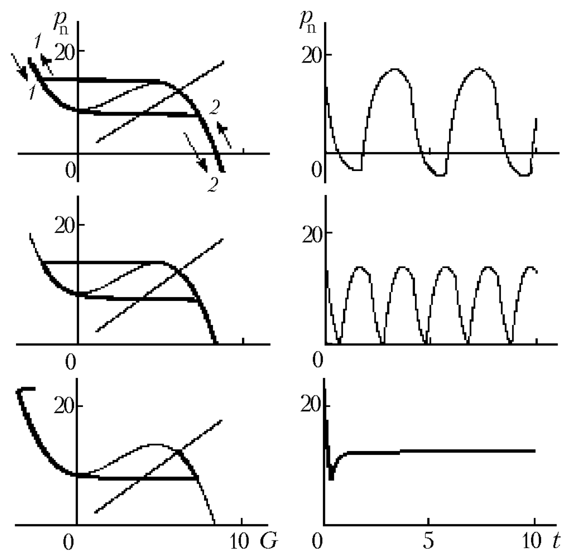

Fig. 5. Reduction in the amplitude of relaxation self-oscillations p_n formed on the descending branch of the characteristic $F(G)$ with decrease in the lag τ and their disappearance for $\tau = 0$. p_n, MPa; G, kg/sec; t, sec.

$F(G)$ that are also the portions of the limiting cycle decrease with τ values. At the same time, the self-oscillation amplitudes which disappear for $\tau = 0$ decrease and the regime becomes stationary.

If we have the movement of the stationary regime on the descending branch of the characteristic $F(G)$ with increase in the flow rate G, self-oscillations are excited only for $\tau \neq 0$. However, when the stationary regime is displaced below a $F(G)$ point at which portion *2–2* of the $F(G)$ plot increasing the limiting cycle is attached to the limiting cycle for $\tau \neq 0$ (Fig. 5), no self-oscillations are excited for any values of the lag τ. Also, no self-oscillations occur for $\tau = 0$ if the entire head characteristic is a monotonically decreasing function, since, according to [10], the presence of the saddle-shaped portion on the head characteristic is necessary for relaxation oscillations which are determined by the periodic solution of the degenerate system of equations. When the operating regime is on the descending branch of the characteristic $F(G)$, the manifestation of the phenomenological lag τ is also necessary for exciting self-oscillations, in accordance with the above substantiations.

Conclusions. It has been established that the amplitude of relaxation self-oscillations formed in the liquid-propellant rocket engine for $L_{a,c.ch} \to 0$ increases with values of the phenomenological lag τ (Fig. 4b and c). Also, such self-oscillations [5–7] occur in the region of the descending branch of the head characteristic $F(G)$ for $\tau \neq 0$ (Fig. 5).

As the mass flow rate G increases, when the point of the operating regime of the liquid-propellant rocket engine moves away from the ascending branch of the head characteristic $F(G)$ and approaches the upper point of portion *2–2* (Fig. 4c), self-oscillations disappear. Thus, the regimes (considered in [5–7]) of unstable combustion with considerable oscillation amplitudes for $L_{a,c.ch} = 0$ and the absence of the ascending branch on the head characteristic $F(G)$ whose existence in the combustion chambers of the liquid-propellant rocket engines remains unknown are not excited for the $\tau = 0$ phenomenological lag of the process of combustion.

NOTATION

$C_{a,c.ch}$, acoustic flexibility of the combustion chamber, m·sec^2; $c(T_n)$, velocity of sound in the flow, m/sec; E, total energy of the flow, J/kg; c_p and c_v, specific heats at constant pressure and at volume, J/(kg·K); G, mass flow rate in the combustion chamber, kg/sec; G_0, stationary mass flow rate in the combustion chamber, kg/sec; G_n, mass flow rate through the nozzle, kg/sec; G_{pr}, propellant flow rate, kg/sec; G_{ox}, oxidizer flow rate, kg/sec; $h_l(G)$, hydraulic loss along the length, MPa; $h_{in}(G)$, hydraulic loss on injectors, MPa; $h_T(G)$, pressure loss by heat supply, MPa; k, adiabatic exponent; l, length, m; $L_{a,c.ch}$, acoustic mass of the combustion chamber, m^{-1}; m, mass of the gas, kg; $p_{c.ch}$, pressure in the combustion chamber, MPa; p_{in}, pressure ahead of injectors, MPa; p_{fr}, pressure ahead of the flame front, MPa; p_n, pressure at entry into the nozzle, MPa; p_{press}, tank-pressurization pressure; q, heat supplied to the flow, J/kg; R, gas constant of the combustion products, J/(kg·K); s, area of normal cross section of the combustion chamber, m^2; s_{cr}, area of critical cross section of the jet nozzle, m^2; t, time, sec; T, absolute temperature, K; u, internal energy, J/kg; u_1 and u_2, internal energy in cross section *1–1* and *2–2*, J/kg; $V_{c.ch}$, volume of the combustion chamber, m^3; w, velocity of motion of the propellant after its ignition, m/sec; Z, wave resistance of the combustion chamber, (m·sec)$^{-1}$; Z^*, wave resistance for which a constant-amplitude limiting cycle is formed, (m·sec)$^{-1}$; β, flow-rate dimensionless number; $\rho_{c.ch}$, density of the combustion products, kg/m^3; ρ_n, density of the combustion products before the entry into the critical cross section of the combustion chamber, kg/m^3; τ, combustion lag, sec; $\varphi(G_n, c(T_n))$, characteristic of the nozzle, MPa. Subscripts and superscripts: a, acoustic quantity; pr, propellant; liq, liquid; cr, critical; c.ch, combustion chamber; ox, oxidizer; f, feed, supply; n, nozzle; fr, ahead of the flame front; v, at constant volume; p, at constant pressure; *, formation of a constant-amplitude limiting cycle; 0, stationary regime; 1, parameters in cross section *1–1*; 2, parameters in cross section *2–2*.

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